Name solutions

ECE 311 Exam 3 Fall 2010

December 1, 2010

Closed Text and Notes, No calculators

1) Be sure you have 11 pages and the additional 5 pages of equations.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) Write neatly, if your writing is illegible then print.

4) This exam is worth 100 points.

(10 pts) 1. A wire lies along the z-axis and carries a current of $\frac{1}{\mu_O}$ A in the +z direction. A second parallel wire passes through x = 1m and carries a current of 2π A in the +z direction. What is the force per unit length on the wire that passes through x = 1 m.

The magnetic field caused by the wire on the z-axis is,

$$\vec{H} = \frac{\left(\frac{1}{\mu_o}\right)}{2\pi\rho}\hat{a}_\phi \frac{A}{m} = \frac{1}{2\pi\mu_o\rho}\hat{a}_\phi \frac{A}{m}$$

$$\vec{B} = \mu_o\vec{H} = \frac{1}{2\pi\rho}\hat{a}_\phi \frac{A}{m}$$

The force on a differential segment of the wire through $x = 1\,m$ is

$$d\vec{F} = I\,d\vec{l} \times \vec{B} = (2\pi)\,dz\,\hat{a}_z \times \frac{1}{2\pi}\hat{a}_\phi \; n$$

$$= -dz\,\hat{a}_x \; n$$

$$\frac{d\vec{F}}{dz} = \text{force per unit lenghth} = -1\,\hat{a}_x \; n$$

(10 pts) 2. Circle true or false concerning the statements for a ferromagnetic material.

If a charged particle is moving in a straight line through a region of space, then the magnetic flux density must be zero.	True	False
Ferromagnetic materials display a linear relationship between B and H.	True	False
In a uniform magnetic field, the net force on a current loop is zero.	True	False
A magnetic force on a current carrying wire is greatest when the wire is parallel to the magnetic field.	True	False
A circular, planar current loop in a uniform magnetic field will rotate till the plane of the current loop is perpendicular to the direction of the magnetic field.	True	False

(7 pts) 3. A 2 C charge is moving with velocity $\mathbf{u}=2\,\hat{\mathbf{a}}_x \frac{\text{m}}{\text{s}}$ in a magnetic flux density of $\mathbf{B}=0.5\hat{\mathbf{a}}_z \frac{\text{Wb}}{\text{m}^2}$.

What electric field would result in this charge moving in a straight line?

$$\vec{F} = Q(\vec{E} + \vec{u} \times \vec{B}) = 0$$

$$\vec{E} = -\vec{u} \times \vec{B} = -(2\hat{a}_x) \times (0.5\hat{a}_z) \frac{m}{s} \frac{Wb}{m^2}$$

$$= -1\hat{a}_x \times \hat{a}_z \frac{Wb}{sm} = -(-\hat{a}_y) \frac{Vs}{sm}$$

$$\boxed{\vec{E} = +1\hat{a}_y \frac{V}{m}}$$

(5 pts) 4. The $\oint \mathbf{E} \cdot d\mathbf{l} = 0$

A) always because **E** is a conservative field

B) only if the magnetic flux is not changing inside the path of integration.

C) only if there is no magnetic field

D) only if the path of integration is a conducting circuit.

(5 pts) 5. The magnetization field saturates in a ferromagnetic material because

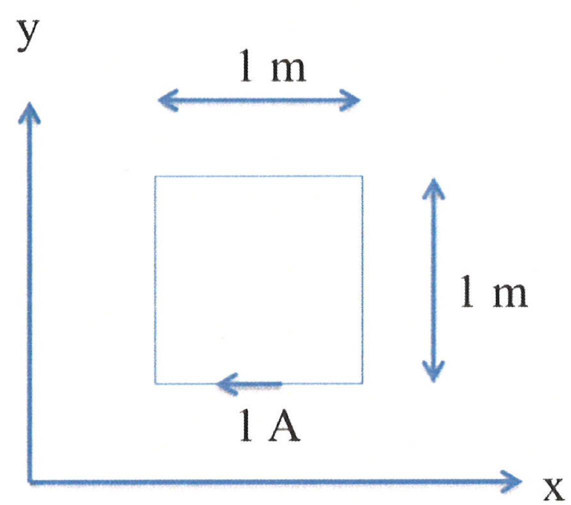
A) all the atomic dipoles are already aligned to the applied magnetic field.

B) thermal energy in the material does not allow any more alignment of the atomic dipoles.

C) of a limit on the size of the applied field.

D) an internal field is generated to oppose the applied field.

(5 pts) 6. A current loop is in the xy plane as shown. If a current of 1 A is flowing in the direction of the arrow, clockwise, what is the magnetic dipole moment of this current loop?

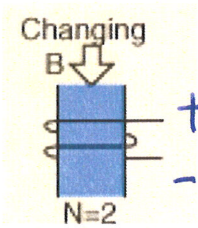


$$\vec{m} = I S \hat{a}_n = (1\,A)(1\,m^2)(-\hat{a}_z)$$

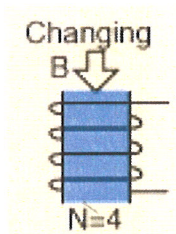
$$\boxed{\vec{m} = -1\,\hat{a}_z \ A m^2}$$

(6 pts) 7. For the following two coils the B field is increasing in the direction of the arrows and is completely inside the coil. Inside the coils pictured, $\frac{d\psi}{dt} = 5\frac{Tm^2}{s}$. Indicate the polarity and value of the voltage that develops across the open ends of the coils.

$$\frac{Tm^2}{s} = \frac{Wb}{m^2}\frac{m^2}{s} = \frac{Wb}{s} = \frac{Vs}{s} = V$$



+ 10V −

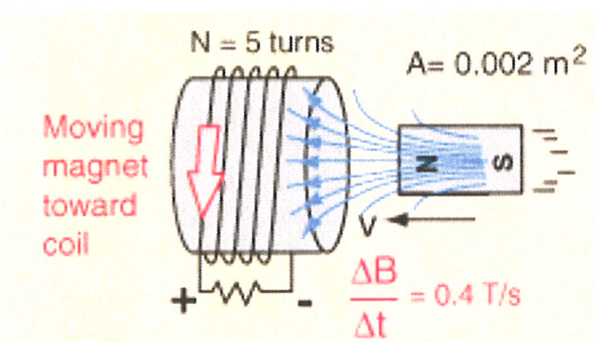


+ 20V −

$$|V_{emf}| = N\frac{d\psi}{dt}$$
$$= 2(5V)$$
$$= 10V$$

$$|V_{emf}| = N\frac{d\psi}{dt}$$
$$= 4(5V)$$
$$= 20V$$

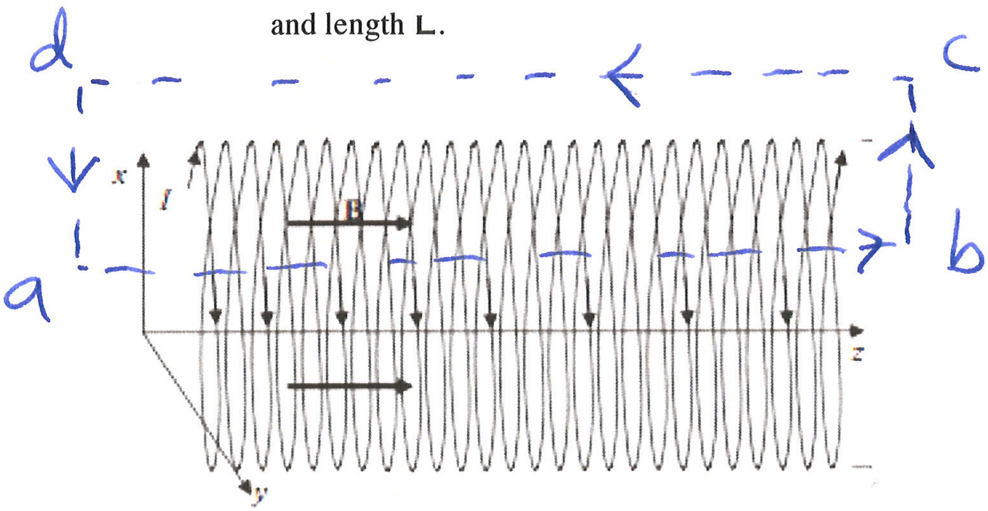
(5 pts) 8. A magnet is moved towards the coil. Assume the magnetic field is uniform in the coil and increasing at a rate of 0.4 T/s. What voltage develops across the resistor?



$$|V_{emf}| = N\frac{d\psi}{dt}$$
$$= N A \frac{dB}{dt}$$
$$= (5)(0.002\,m^2)(0.4\,\frac{T}{s})$$
$$= 0.004\,\frac{Tm^2}{s}$$
$$= 0.004\,V$$

$$\frac{Tm^2}{s} = \frac{Wb}{m^2}\frac{m^2}{s} = \frac{Wb}{s} = V$$

(15 pts) 9. Determine the inductance for the N turn tightly wound solenoid shown of cross-sectional area S and length L.



We need to find the magnetic field inside the coil. Apply Ampere's circuital law to the dashed path

$$\oint \vec{H}\cdot d\vec{l} = NI = \int_a^b \vec{H}\cdot d\vec{l} + \int_b^c \vec{H}\cdot d\vec{l} + \int_c^d \vec{H}\cdot d\vec{l} + \int_d^a \vec{H}\cdot d\vec{l}$$

The field is weak from $c \to d$ and $\vec{H}$ is essentially perpendicular to $d\vec{l}$ along $b \to c$ and $d \to a$

so $\oint \vec{H}\cdot d\vec{l} \approx \int_a^b \vec{H}\cdot d\vec{l} = NI$

$\vec{H}$ must be constant inside the coil since $\oint \vec{H}\cdot d\vec{l} \approx \int_a^b \vec{H}\cdot d\vec{l}$ must be the same wherever we go through the coil so,

$$H\ell = NI \Rightarrow B = \mu_0 H = \frac{\mu_0 N I}{\ell}$$

$$\phi = \int \vec{B}\cdot d\vec{s} = \frac{\mu_0 N I}{\ell} S$$

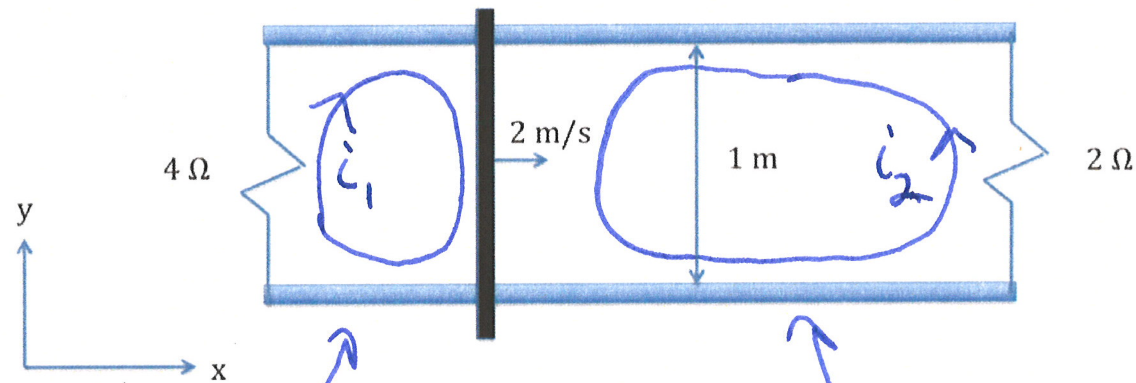
$$L = \frac{\lambda}{I} = \frac{N\phi}{I} = \frac{N}{I}\frac{\mu_0 N I}{\ell} S$$

$$L = \frac{\mu_0 N^2 S}{\ell}$$

(14 pts) 10. A conducting bar slides on two parallel rails as shown. Across both ends of the rails are resistors as shown. The rail system is in the xy-plane and everywhere is a uniform magnetic flux density of $\mathbf{B}=0.5\hat{\mathbf{a}}_z \frac{\text{Wb}}{\text{m}^2}$. The bar is moving with velocity, $\mathbf{u}=2\frac{\text{m}}{\text{s}}\hat{\mathbf{a}}_x$.

(4 pts) a) If currents are flowing, indicate on the figure the direction of these currents.

(10 pts) b) Determine the values of the currents that are flowing.



$$\frac{dA}{dt} = 2\frac{m^2}{s}$$

$$\frac{d\psi}{dt} = B\frac{dA}{dt}$$

$$= \left(0.5\frac{Wb}{m^2}\right)\left(2\frac{m^2}{s}\right)$$

$$= 1\frac{Wb}{s} = 1V$$

$$i_1 = \frac{1V}{4\Omega} = 0.25A$$

$$\frac{dA}{dt} = -2\frac{m^2}{s}$$

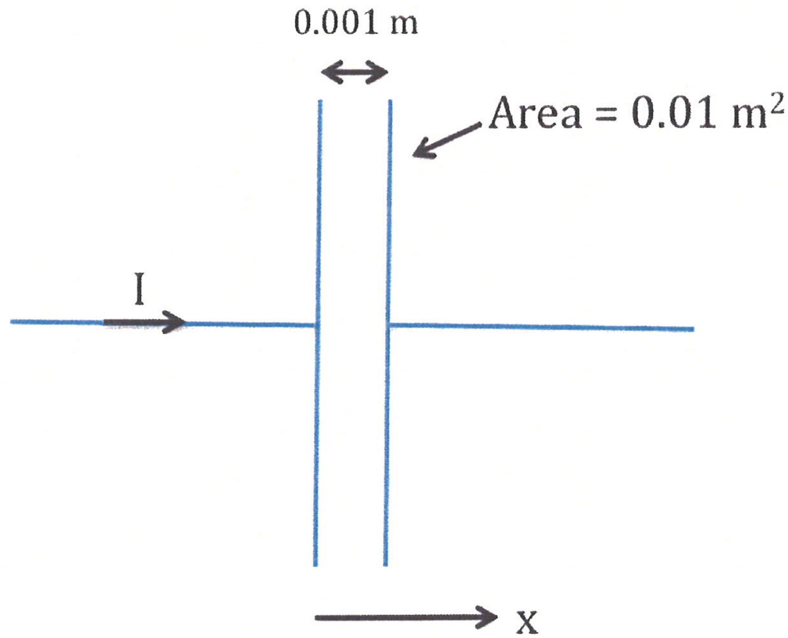
$$\frac{d\psi}{dt} = B\frac{dA}{dt}$$

$$= \left(0.5\frac{Wb}{m^2}\right)\left(-2\frac{m^2}{s}\right)$$

$$= -1\frac{Wb}{s} = -1V$$

$$i_2 = \frac{1V}{2\Omega} = 0.5A$$

(6 pts) 11. The following capacitor is being charged by a current of 2 A.



(4 pts) a) What is the displacement current and the displacement current density between the plates of the capacitor?

$I_d = 2A$ $\quad J_d = \frac{I_d}{A} = \frac{2A}{0.01 m^2} = 200 \frac{A}{m^2}$

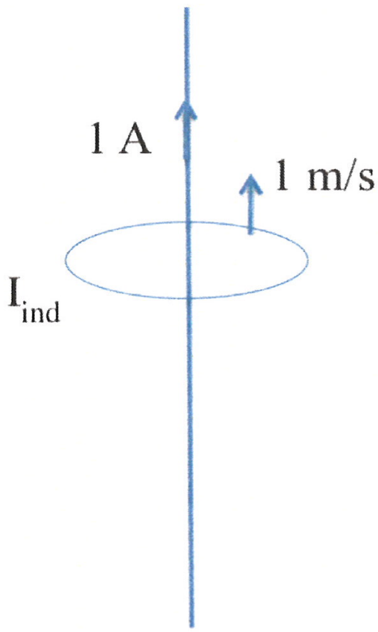
$\vec{J}_d = 200 \frac{A}{m^2} \hat{a}_x$

(2 pts) b) If the plate separation was changed to 0.002 m, how would the displacement current in the capacitor change? Note I remains at 2 A.

it would not change

(12 pts) 12. For each of the following structures circle the correct answer. The vertical wires are infinite in length and carry a current of 1A in the direction of the arrow on the wire.

a) The loop is moving relative to the wire as indicated

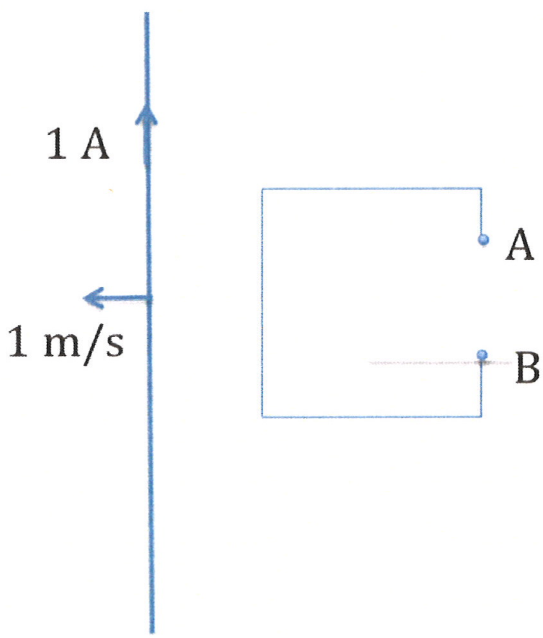


$I_{ind} > 0$

$I_{ind} = 0$

$I_{ind} < 0$

b) The wire is moving relative to the open loop as indicated

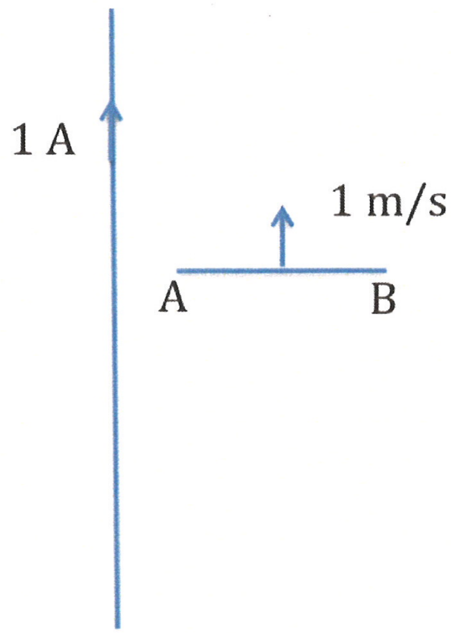


$V_A > V_B$

$V_A = V_B$

$V_A < V_B$

b) The short wire, from A to B, is moving relative to the infinite wire as indicated



$V_A > V_B$

$V_A = V_B$

$V_A < V_B$